Multidisciplinary Design Optimization Under Uncertainty: An Information Model Approach

Abstract

Motivated by needs of concurrent multi-disciplinary design of a multi-purpose vehicle, a modeling and methodological approach to handling tradeoffs is presented. Each component has uncertain elements and a random performance which is influenced by the performance of other components. The components may require different knowledge bases and models with different mathematical structures, time and size scales, calling for higher-level coordination.

The theory of reproducing kernel Hilbert spaces provides the mathematical foundation for the approach. Performance is modeled as a random function of uncertainties that are considered as independent variables. Higher-level design decisions, the result of tradeoffs between alternative component designs, are based on information models of component performance functions. The models make use of second-order statistics of the performances and an algebra of their reduced-order representations. Multicriteria optimization methods are used to determine preferred overall designs.

1   Introduction

Challenges associated with decision making for large complex systems in the presence of uncertainty and risk have been of special interest to scientists and engineers in many disciplines. In engineering design, the consistent effort to come up with methodologies in support of complex systems design has been reflected since the nineteen eighties in the creation and development of multidisciplinary design optimization (MDO). As a consequence, a dominating majority of studies on complex systems in engineering design has been based on mathematical optimization and, more specifically, mathematical programming. The inclusion of uncertainty has made the complex system modeling more realistic because it recognized the inability to determine the true state of affairs of a system, but also it has made the modeling more difficult due to the challenge of representing the unknown. In effect, the main research effort has evolved in two concurrent directions of more effective optimization formulations and more sophisticated models of uncertainty.

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A number of mathematical concepts and theories have been used to represent aleatory and epistemic uncertainties (Helton and Oberkampf, 2004) and put them to work in concert with MDO. Under the assumption that every uncertainty has a distribution, which is amenable to Bayesian techniques (Berger, 1985) and favored by the largest part of the operations research community as discussed in Rockafellar (2007), probability theory with random variables gave rise to reliability-based design optimization (RBDO). Possibility theory with fuzzy variables and possibility distributions, and evidence theory with measures of belief and plausibility have been used to relax stronger requirements of probabilistic methods (see e.g., Kohlas and Monney, 1995). Refer to Agarwal et al. (2004), Youn et al. (2004), Chiralaksanakul and Mahadevan (2007), Zhang and Huang (2010), Zhao et al. (2010), and others for MDO-based approaches and Du and Chen (2005), Wang et al. (2009), and Huang et al. (2010) specifically for collaborative optimization-based methods that make use of these theories. MDO, originally based on single objective optimization, has also used multiobjective optimization (Li and Azarm, 2008). During the last decade, analytical target cascading (ATC), an optimization methodology for hierarchical decomposition and coordination of complex systems in enginering design, has developed independently of MDO. It has been extended to probabilistic ATC to account for aleatory uncertainty (Kokkolaras et al., 2006; Han and Papalambros, 2010; and Xiong et al., 2010). Game theory (see e.g., Gupta and Krishnamurthy, 2004), rough sets theory (see e.g., Shao et al., 2008), or kriging models (see e.g., Martin and Simpson, 2006) are examples of other mathematical tools that have been selected to model uncertainty. To represent uncertain variables for which no information is available, interval analysis has been applied (Du and Zhang, 2010). Some authors employ various combinations of these mathematical tools in a quest to propose more effective methodologies supporting design (see e.g., Guo and Du, 2010). To recognize research efforts beyond optimization, we refer to Gurnani and Lewis (2004) where the choice of a preferred design results from a selection process from a finite set of alternatives.

Despite many advances the design of complex engineered systems remains a challenge and novel modeling and methodological paradigms are called for. Modeling and decision difficulties for complex systems are highlighted in the U.S. Department of Energy report of Hendrickson and Wright (2006). For instance, when a design problem is decomposed, the components can require different knowledge bases. Some components may have dynamic physics-based models with force or energy contraints and others static utility-based models with geometric constraints (Brown et al., 2008, p.11). Also, the components might have radically different time and size scales (Brown et al,. 2008, p. 12). If we do not decompose, an all-at-once approach to optimization will be complicated by the different mathematical representations of the components (see Brown et al., 2008, p. 15). The NSF report of Simpson and Martins (2010) contains recommendations for MDO to advance the design of complex engineered systems while the United States Army Research Laboratory Broad Agency Announcement in Mathematical Sciences (2010) calls for the development of quantitative models of complex phenomena.

Our research is heavily influenced by the government reports cited. Of special interest to us are the challenges associated with the decision making under uncertainty and risk, which are present in vehicle design, particularly in the concurrent design of vehicle components requiring disjoint knowledge bases. These multidisciplinary design problems raise questions of communication across discipline boundaries, appropriate tradeoffs, and achieving total system goals. Our research is illustrated by the need to coordinate the concurrent design of a multi-purpose vehicle suspension system operating in an uncertain environment with random exogenous forcing, and the design of the vehicle payload space with an uncertain mission and random performance. While the two design problems require different knowledge bases and models with different mathematical structures, time scales, and size scales, the design decisions must be coordinated because of design interactions between the two systems. Imposition of *a priori* requirements to insure compatibility by reducing the design space of the two design problems might result in a vehicle with a restricted range of operating environments or reduced mission capability.

We present an approach to handling tradeoffs in a multi-discipline design problem, where the design model for each discipline has uncertainty and random performance which may also be influenced by the performance of one or more of the other discipline design models. The approach is illustrated on a small computational example with difficulties similar to those present in the full vehicle design problem. One component design problem is a suspension design balancing the total load (empty vehicle, cargo, and passengers) which reduces to a component decision problem using a physics-based stochastic dynamic response model (a stochastic differential equation (SDE) model) with one design variable and two uncertain environmental variables (Kloeden and Platen, 1992). The problem is formulated as a selection problem minimizing the measured system imbalance over a finite set of environments and designs. The other component design problem is a payload capacity design with the the goal of a division of payload space between cargo and passengers and the maximization of a random utility function. This design problem reduces to a component decision problem formulated as a utility-based stochastic portfolio problem with a single uncertain variable modeling the vehicle's mission. In this two-level multicriteria decision process, the lower-level optimization problems produce multiple candidate subdesigns that are efficient at the lower level, while the higher-level design selection problem accounts for interactions between the lower-level efficient alternatives and results in a preferred design over the full range of uncertain operating environments and missions.

The paper is structured is follows. In Section 2, we discuss particular aspects of our approach including decision making under uncertainty and risk, information models, the design context, reproducing kernel Hilbert spaces, and separable random fields. In Section 3, a "toy" vehicle design problem with uncertainty and risk is presented as a simplified but convincing example. The example illustrates a two-level design process with multiple decision makers using a multicomponent information model. Section 4 introduces a simple computational example with re-

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duced order information models of the components. An approach to handling a multiple-component design problem by constructing a higher-level decision model is given in Section 5. Section 6 concludes the paper.

2   Particular aspects of the problem

Uncertainty, risk and rational decision making.   Oberkampf et al. (2004) produced a very useful summary of attitudes toward uncertainty in the engineering community. Uncertain variables are classified as aleatory if they have a distribution and as epistemic if they do not have a distribution. For instance, future vehicle operating conditions or vehicle missions will have distributions from a fleet perspective but lack distributions at the individual vehicle level. In the design process we can not predict with confidence the operating conditions and mission of any individual vehicle. Thus we must design for a range of operating conditions and missions, i.e., treat operating conditions and missions as epistemic uncertainties. This example shows that the distinction between aleatory and epistemic variables is really about the modeler's perspective rather than properties of the variable. From a bird's viewpoint a variable will be aleatory while from a frog's viewpoint the variable will be epistemic. While many traditional decision methodologies have taken the simpler bird's viewpoint, the frog's viewpoint may be required for a better decision over a range of uncertainty values. In our approach, uncertainty is system variability that does not have a probability distribution, first introduced by Knight (1921). Our uncertainties without qualifiers are epistemic and modeled as independent variables defined on subintervals of $\mathcal{R}^n$ (Reneke and Samson, 2008). In Samson et al., (2009), this approach is presented against a review of different perspectives on uncertainty and risk.

With the increase in the complexity and magnitude of decision problems, the rationality of the decision process has come to the spotlight (Hazelrigg, 2003). Savage (1972) outlines conditions for preferences in his Postulates I-IV to ensure rational decision making. Our approach to decision making satisfies the Savage postulates. Multicriteria optimization (Ehrgott, 2005), a powerful methodology when extended to problems where criteria values are functions of the uncertainties, enables us to verify that decisions are rational.

MDO and information models.   Within the MDO framework, discipline-based decomposition leads to radically different models of components based on different knowledge domains (see e.g., Du and Gunzberger, 2000). Aside from the questions of uncertainty and risk, the components interact through the exchange of physical quantities rather than through the exchange of information, which is a fundamental difference from our modeling approach. The MDO system model is still one large physics-based model. The decomposition does reduce the computational burden of the optimization problem but does not reformulate the model reducing the order of the model. In our approach, information models are used to represent

components interacting through the exchange of information and not subject to conservation rules, and also to reduce the order of the model. Furthermore, we do not use mathematical programming in the traditional sense but optimize over a finite set of alternatives.

Design stages and levels, and problem decomposition. Reneke and Wiecek (2005) presented a vision for the design of complex systems and now make use of that earlier language in this exposition. In order to properly understand the context for our higher-level design problem we introduce a design framework. We assume that design in complex systems proceeds through stages from preliminary designs to final designs. At each stage decisions will have been made in previous stages which are taken as constraints in the current stage.

Within each stage the basic strategy is problem decomposition from the top down and decision making from the bottom up. In early stages the decomposition will tend to focus on function, for instance, vehicle configuration. In middle stages the decomposition will tend to focus on system component design within the constraints set from earlier design stages. In later stages design alternatives will have narrowed and constraints tightened leading to a final preferred design. As the decomposition proceeds, more and more specialized knowledge, including knowlege of uncertainties and risk, is required leading to the introduction of design levels. The design levels represent different knowledge bases (different knowledge not more) required for coordinating component designs passed up from the next lower level. For a discussion of a top down decision strategy see Butler (1977) and Anandalingam (1988).

Design stages support the orderly introduction of system requirements. Ideally, requirements are not introduced too early limiting the design space or too late forcing costly design fixes. Design levels support orderly tradeoffs between lower-level alternatives based on higher-level objectives. Design stages imply an iterative design process with higher-level decision makers learning of lower-level design possibilities and lower-level decision makers learning of higher-level design objectives.

Reproducing kernel Hilbert spaces. Infinite dimensional function spaces provide the natural setting for design problems with uncertainty in Knight's sense. We make several modeling assumptions that specialize the function spaces reducing the design problems to tractable decision problems. Keep in mind the two spaces that we must provide for, the space of random fields modeling stochastic system performance and the space of deterministic decision variables. The fundamental connection is provided by reproducing kernel Hilbert (RKH) spaces, an old topic dating from the first half of the twentieth century with many contributors and wide applications (Stone, 1932; Aronszajn, 1950). More narrowly, our subject begins with the realization that covariances are nonnegative definite functions (Loève, 1948) establishing a relation between probabilistic models and reproducing kernel Hilbert spaces.

We have pursued a path exploring special cases but with accessible results (Reneke and Samson, 2008). The covariance kernel of the standard Wiener process on $[0, 1]$ is the reproducing kernel of the space G of Hellinger integrable functions g on $[0, 1]$ with $g(0) = 0$ introduced in Hellinger (1907). The covariance R of a general zero mean Gaussian process $\{\hat{X}_t, 0 \le t \le 1\}$, subject to conditions, is the matrix representation of a nonnegative definite operator on the complete inner product space of Hellinger integrable functions on $[0, 1]$. Further, discretizations $\{\hat{X}(t(k)), 0 = t_0 < t_1 < \ldots < t_n = 1\}$ can be simulated by $\{[(R_{tt}^C)^T Z_t](k)\}_{k=0}^n$, where R_{tt}^C is the upper generalized Choleski factor of an $(n+1) \times (n+1)$ nonnegative matrix R_{tt} and Z_t is an $(n + 1)$-dimensional $(0, 1)$-normal random vector.

Our spaces, operators, and representations fit into a framework provided by the general theory due to Mac Nerney (1980). Since the finite dimensional positive definite matrices will be discrete approximations of positive definite linear operators, the decision maker will not need to go beyond the discrete representations.

We are concerned only with a special class of random fields that can be analyzed in terms of marginal distributions. The simplification permits us to extend the analysis of random processes to random fields.

Separable representations. Our development of a class of zero mean Gaussian random fields follows the pattern of the Wiener field W on the unit cube $[0, 1] \times [0, 1] \times [0, 1]$ (Chentsov, 1956). In particular, the covariance kernel of W has a simple form, namely,

$$E(W(a, b, c)W(t, u, v)) = min(a, t)min(b, u)min(c, v).$$

This property generalizes for certain zero mean random fields $\hat{X}$ to

$$
\begin{aligned}
E(\hat{X}(a, b, c)\hat{X}(t, u, v)) &= R(a, b, c, t, u, v) \\
&= R_1(a, t)R_2(b, u)R_3(c, v),
\end{aligned}
$$

where R is the covariance kernel of $\hat{X}$ and each R_i is a nonnegative definite function on $[0, 1] \times [0, 1]$. Fields with this property are said to be *separable* (Vanmarcke, 1983). In subsequent sections we will show that discretizations of $\hat{X}$ can be simulated in a manner analogous to the simulation of the discrete random process described above, a key to our information models. Further, reduced order representations of separable fields $\hat{X}$ and $\hat{Y}$ will be available for modeling interactions between system components with response fields $\hat{X}$ and $\hat{Y}$. Refer to Gohberg and Krein (1970) for a classic development of the operator algebra for a one variable deterministic case.

In general, the independence expressed in the factorization condition above might not hold. However, such independence is implicit in the common engineering practice of exploring a given physical system by allowing only one quantity to vary at a time.

Vanmarcke (1983) provides a foundation reference for separable fields in engineering with separability given in terms of correlation functions rather than covariance kernels. Refer to Yaglom (1986) for using correlations rather than covariances

and alternative representations for random fields. While related to our random fields, random space-time functions have a different flavor (Kyriakidis and Journel, 1999; Genton, 2007). In fact, our random fields would correspond to the limited case where spatial behavior is constant with respect to time.

3 A "toy" vehicle design problem with uncertainty and risk

In this section, a simple illustrative example is introduced that infuses the modeling/decision development with needed realism and confronts us with modeling difficulties that must be resolved for the decision approach to be applicable to more complex design problems. However, the approach is more general than the example and the particulars of the example only illustrate the range of possible applications. A preview of the component computational models is given in general terms. The component models illustrate concretely issues that must be resolved in order to construct a multicomponent model. Finally, information models are discussed in general terms in preparation for the construction of the higher-level model used to coordinate the component designs, i.e., choose from alternative component designs.

3.1 Introduction of the simplified example

Our task is developing a methodology for handling a multiple component/multiple discipline design problem. In this paper we are limited to a two-level design process with two lower-level designers from different disciplines using models with incompatible mathematical formulations.

The higher-level design goal for the example is a low maintenance/high utility vehicle. The higher-level vehicle design problem decomposes into separate lower-level design problems for the vehicle suspension and partition of the payload space between passengers and cargo. Each of the component design problems involve uncertainty and risk. The lower-level problems produce alternative designs that result in a single preferred design at the higher level taking into account tradeoffs of various combinations of the lower-level designs. The example produces a suspension system design and a logistics design that result in a low maintenance/high utility vehicle that minimizes risk over the range of uncertainty values.

The two-component design problems use distinctly different mathematics and methods. The two submodels represent assumptions about the objective world that neither set of designers would feel comfortable making for the other component, i.e., the logistics designers cannot easily become suspension designers and similarly for the suspension designers. Small changes in the underlying assumptions can lead to significantly different preferred designs.

Both component problems are of high dimension because of the uncertainties and so directly combining the two models seeking a single preferred design would be

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infeasible. The component problems will be simplified so the difficulties inherent in multidisciplinary optimization are not masked. Further, the submodel descriptions will be straight forward and the computational burden light enough that an overall system picture can emerge.

3.1.1 The conceptual design problem

Every design problem has a context. In our idealized scenario we assume that the vehicle design proceeds through stages starting with a preliminary design stage and ending with a final design stage resulting in a preferred vehicle design. Decisions will have been made in previous stages and are treated in the current stage as constraints. For our "toy" problem we consider a late stage problem. The model constants, maximum payload weight and available space for passengers and cargo, are set.

Within each design stage the vehicle design problem is decomposed into design levels with multiple decision makers at each level. Decomposition proceeds from the top down and decisions from the bottom up. Knowledge and design goals are not uniformly distributed across levels but rather are stratified. Adjacent levels and different components at a given level represent different knowledge bases. We also can expect different tolerances for risk and different design uncertainties between levels and components. The "toy" problem will illustrate these difficulties and provide a platform for testing our ideas for dealing with these difficulties and others that will emerge.

The "toy" problem is presented as a problem of coordinating two lowest-level designs. The lowest level is not at the finest granularity (Du and Gunzburger, 2000) and so the two component models do not interact physically, i.e., by exchanging material, forces, or energy with conservtion laws. Different size and time scales would make interactions difficult. Rather the components interact through an exchange of information which is modeled by the higher-level decision maker.

Unmodeled uncertainties and criteria are passed down from the higher level. At the lower level uncertainties are interpreted in terms of the lower level models and modeled as number intervals. Component performances of selected designs are modeled as random functions of the uncertainties and the second order statistics of the performances, again functions of the uncertainties, are passed up to the next higher level.

Uncertainties, criteria, and design performances received by the higher level also have to be interpreted. Balance as measured by the suspension designer will be interpreted by the higher-level decision maker as vehicle stress with implications for maintenance. Similarly, payload utility as measured by the logistics designer will be interpreted as vehicle usefulness.

In general terms, the uncertain vehicle operating environment, important for the suspension system design, is modeled as uncertain parameters at the suspension design level. The uncertain vehicle mission, important for the logistics design, is modeled as the average trip length at the logistics design level. Random variability

in performance results from exogenous random forces on the suspension system, depending on the operating environment, and background (exogenous) conditions, depending on the mission, affecting the performance of the payload design. The designer must plan for the random disturbances but the exogenous "noise" is outside his/her control.

3.1.2   Higher-level component interactions based on information

Remember that our modeling proceeds from the top/down. Further, the higher-level modeler does not know or understand the lower-level models. The diagram in Figure 1 represents the higher-level conceptual model of the interactions of the two lower-level components. The arrows are "influences", the W_i's are exogenous disturbances, and the G_i's are component responses. The details will be filled in as we proceed with our exposition.

The conceptual model is an assumption about the interactions of component performances. The zero mean random part of the suspension balance as information will be taken as part of the background "noise" for the payload utility. The zero mean random part of the payload utility as information will be taken as part of the background noise for the suspension balance.

To understand component performances as information, consider the following. If $\equiv$ stands for surrogate then 'balance of suspension design' $\equiv$ 'low vehicle and payload stress' $\equiv$ 'low maintenance'. Further, 'payload utility' $\equiv$ 'vehicle payload variability' $\equiv$ 'high maintenance.' Thus the higher-level decision maker must consider tradeoffs of vehicle usefulness and vehicle maintainability.

3.2   Preview of the information model

Physics based models typically are concerned with flows of material, forces, or energy all subject to conservation laws. In contrast, information from different sources affecting a component's performance is summed (linear combinations). Information of the performance of a given component can be shared equally by any number of additional components.

Statistics is the art and science of extracting information from data sets. Statistical models, particularly correlation models, are higher-level or information models. Examples of where information models have been useful include models for life cycle performance, models for distributed manufacture, econometric models, management models, finance models, and security models. Our concern is for information models based on second-order statistics using an algebra of linear operators on classes of functions of the underlying uncertainties.

For vehicles, the component design problems use physics-based models, but the higher level coordination of lower level designs is based on information models. The models enable the decision maker to take into account the influences component designs have on one another. As the decision process moves up a level information is lost. This is a familiar phenomenon in statistical models. The art enters through

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the selection of features of the lower-level models that are important to the higher-level decisions. This selection process is carried out by the lower-level designers. For instance, "balance" for the lower-level suspension designer will have an operational definition which is lost for the higher-level decision maker but is acceptable to the lower-level designer as a surrogate for future suspension maintenance.

Outline of information model requirements. Higher-level information models are based on second-order statistics of component performances as functions of the uncertainties. In the modeling phase, criteria and uncertainties will be passed down to the component designers. The criteria and uncertainties have to be interpreted by the lower-level designers in terms of the lower-level models. Performances of the lower-level designs are passed up in terms of the second-order statistics. The statistics are the only information on the lower-level designs available to the higher-level designer.

For instance, the higher-level designer is interested in a low-maintenance vehicle operating in a range of conditions. The criterion that is passed down has to be meaningful to the lower-level designer in the current design stage. Maintenance costs as a criterion may be meaningless since external factors influencing costs may lie outside the lower-level designer's purview. Both designers can agree on a measure of suspension balance as a criterion. The higher-level designer can interpret better balance as a reduction in stress on the vehicle and payload reducing maintenance. The lower-level designer can interpret better balance as a more equal dissipation of energy among the suspension support points increasing the time to first failure.

The information models will be limits of finite discrete approximations. In order to simplify the notation, we adopt the following naming convention for discretizations of functions defined on intervals. Suppose that $\{u_\ell\}_{\ell=0}^L$ is a partition of $[0, 1]$, i.e., $0 = u_0 < u_1 < \ldots < u_L = 1$. If F is a function defined on $[0, 1]$ then F_u is the discrete function or vector given by $F_u(\ell) = F(u_\ell)$. Similarly, for G_v and H_t, where $\{v_m\}_{m=0}^M$ and $\{t_n\}_{n=0}^N$ are partitions of $[0, 1]$. If H is a function of two variables then $H_{uu}(\ell, \bar{\ell}) = H(u_\ell, u_{\bar{\ell}})$ and similarly for H_{vv} and H_{tt}. We take $H_{uv}(\ell, m) = H(u_\ell, v_m)$. If H is a function of three variables then we take $H_{uvt}(\ell, m, n) = H(u_\ell, v_m, t_n)$.

In design problems the emphasis is on component performance fields which are random. Since the higher-level decision models and decisions are based on the second-order statistics of the performances, reduced-order simulations of performances become the immediate objective. We begin with discrete simulations of the Wiener field W defined on $[0, 1] \times [0, 1] \times [0, 1]$, setting the pattern for our simulations of more general zero mean fields. The covariance kernel of W is

$$E(W(u, v, t)W(\bar{u}, \bar{v}, \bar{t})) = min(u, \bar{u}) \cdot min(v, \bar{v}) \cdot min(t, \bar{t}).$$

Let $K(x, y) = min(x, y)$ for $0 \leq x, y \leq 1$. Thus we can construct discrete simulations of W by

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$$W_{uvt}(\ell,\, m,\, n)$$

$$= \sum_{i=1}^{\ell}\sum_{j=1}^{m}\sum_{k=1}^{n}[(K_{uu}^{C})^{T}](\ell,\, i)[(K_{vv}^{C})^{T}](m,\, j)[(K_{tt}^{C})^{T}](n,\, k)Z_{uvt}(i,\, j,\, k) \quad (1)$$

$$= [(K_{uu}^{C})^{T}(\sum_{k=1}^{n}[(K_{tt}^{C})^{T}](n,\, k)Z_{uvt}(\cdot,\, \cdot,\, k))K_{vv}^{C}](\ell,\, m),$$

where Z is a $(0,\, 1)$-normal random function on $[0,\, 1] \times [0,\, 1] \times [0,\, 1]$. In the limit (the limit in distribution),

$$W(u,\, v,\, t) = \int_{0}^{u}\int_{0}^{v}\int_{0}^{t} Z(x,\, y,\, z)\,(dz)^{1/2}\,(dy)^{1/2}\,(dx)^{1/2}.$$

Note that $W(u_{\ell},\, v_{m},\, t_{n})$ and $W_{uvt}(\ell,\, m,\, n)$, the discrete simulation, are both zero mean normal random variables with the same variance and so $W_{uvt}(\ell,\, m,\, n)$ may be thought of as a simulation of $W(u_{\ell},\, v_{m},\, t_{n})$. The reader should also observe that the simulation of $W(u_{\ell},\, v_{m},\, t_{n})$ was constructed using a transformation of the discrete fields Z_{uvt} providing an equivalence between the simulation and the transformation representation of W_{uvt}. An information model of W_{uvt} can be taken in either sense.

A similar construction holds for two parameter Wiener field and the one parameter Wiener process. We will denote all cases with the reserved designation W depending on the context to identify the number of parameters.

Briefly, the technical requirements for constructing an information model are as follows. Given a zero-mean random field $\hat{X}$ on $[0,\, 1] \times [0,\, 1] \times [0,\, 1]$ from a component response, we require the existence of the integral

$$\hat{Y}(u,\, v,\, t) = \int_{0}^{u}\int_{0}^{v}\int_{0}^{t} \hat{X}(x,\, y,\, z)\,(dz)^{1/2}\,(dy)^{1/2}\,(dx)^{1/2}$$

for all $(u,\, v,\, t)$ in $[0,\, 1] \times [0,\, 1] \times [0,\, 1]$. For the existence, we will rely on an application of the Central Limit Theorem, i.e., $\hat{X}$ must satisfy certain conditions. Without further comment, in the illustrative example the conditions hold when required.

We do not need to simulate $\hat{Y}$ but observe that if $E(\hat{X}(x,\, y,\, z)\hat{X}(\bar{x},\, \bar{y},\, \bar{z})) = 0$, when $(x,\, y,\, z) \neq (\bar{x},\, \bar{y},\, \bar{z})$, and if $\hat{X}$ is separable with covariance kernel $R_{1}(x,\, \bar{x})R_{2}(y,\, \bar{y})R_{3}(z,\, \bar{z})$, then the covariance kernel of $\hat{Y}$ is given by

$$E(\hat{Y}(u,\, v,\, t)\hat{Y}(\bar{u},\, \bar{v},\, \bar{t}))$$

$$= \int_{0}^{min(u,\,\bar{u})} R_{1}(x,\, x)\,dx \cdot$$

$$\int_{0}^{min(v,\,\bar{v})} R_{2}(y,\, y)\,dy \cdot \int_{0}^{min(t,\,\bar{t})} R_{3}(z,\, z)\,dz.$$

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Given the covariance kernel $\hat{Y}$ we require that it is the matrix representation of a nonnegative definite operator A on the space of Hellinger integrable functions $g \in G$ defined on $[0, 1] \times [0, 1] \times [0, 1]$, with $g(0, v, t) = g(u, 0, t) = g(u, v, 0) = 0$. If $\hat{Y}$ is separable then there are implications for the operator A. In particular, the field $A^{1/2}W$ defined as a limit using the Central Limit Theorem is a reduced-order representation of the zero mean field $\hat{Y}$, i.e., $\hat{Y}$ and $A^{1/2}W$ have the same covariances (Reneke and Sundeep, 2008). Note that only the second-order statistic, the covariance of $\hat{Y}$, is passed up to the higher-level decision maker. The levels represent different knowledge bases and there is no requirement that the higher-level decision maker understands the models used by the lower-level designer.

4   A simple computational example

The engineering context reduces the suspension design problem to balancing the total load (empty vehicle, cargo, and passengers) using a physics-based dynamic response model (SDE model) with uncertain parameters. The suspension system is modeled as a spring system in the spirit of the classroom approach, as illustrated in Figure 2 (Blundell and Harty, 2004). The payload capacity design problem is similarly reduced to a portfolio problem based on a static stochastic model with uncertain parameters. While each of these component models are well known, we are concerned with their interaction in a simple system in the presence of uncertainty and risk.

In this section, a stochastic 3-spring model of the vehicle suspension will be introduced with two uncertain parameters modeling the uncertain operating environment and one design parameter. The approach to the suspension design in the presence of uncertainties is data-based, i.e., it is based on the toy-problem simulations of the SDE model. A surrogate for system balance will be developed leading to six alternative suspension designs. Mission dependent distributions for the stochastic utilities in the logistics model will be introduced and the approach to the logistic design in the presence of uncertainty differs radically from the design approach used for the suspension system. The approach will lead to twelve alternative designs.

The higher-level designer must coordinate the lower-level designs but need not be aware of or understand the models and methods employed by the lower-level component designers. Performances of the seventy-two different vehicle designs will be evaluated at the higher level in four operating environments with seven missions using a reduced-order vehicle information model. The vehicle model will be introduced in Section 5 when the technical details can be explored.

4.1   The 3-spring model

The deterministic model for the 3-spring system (Reneke et al., 2010) is

$$m_1 y_1'' = -(k_1 + k_2)y_1 - b_1 y_1' + k_2 y_2 + f_1(t)$$

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$$m_2 y_2'' \quad = \quad k_2 y_1 - (k_2 + k_3) y_2 - b_2 y_2' + f_2(t).$$

The spring constants k_1, k_2 and k_3 are fixed for the current design stage. Feasible designs are given by masses m_1, m_2. The friction coefficients b_1, b_2 will be uncertain. Exogenous influences are represented by known time-dependent forces $f_1(t)$ and $f_2(t)$. For simplicity we assume that $f_2 = 0$, $df_1 = -f_1\, dt$, and $f_1(0) = 1$. System output is the measured displacements $y_1(t)$ and $y_2(t)$ of the masses.

Stochastic variability in the spring problem. The deterministic vector ODE model with stochastic forcing becomes the stochastic SDE model

$$dY = FY\, dt + GY\, dW,$$

where

$$F = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ -\frac{k_1+k_2}{m_1} & -\frac{b_1}{m_1} & \frac{k_2}{m_1} & 0 & \frac{1}{m_1} \\ 0 & 0 & 0 & 1 & 0 \\ \frac{k_2}{m_2} & 0 & -\frac{k_2+k_3}{m_2} & -\frac{b_2}{m_2} & 0 \\ 0 & 0 & 0 & 0 & -1 \end{bmatrix}$$

and

$$GY\, dW = \begin{bmatrix} 0 \\ \epsilon_2 Y_2\, dW_2 \\ 0 \\ \epsilon_4 Y_4\, dW_4 \\ \epsilon_5\, dW_5 \end{bmatrix}.$$

We assume that the noise coefficients $\epsilon_2 = 1/2$, $\epsilon_4 = 1/3$, and $\epsilon_5 = 0.1$ are given. The terms dW_2, dW_4, and dW_5 represent exogenous system disturbances. The initial condition $Y(0) = [-1, 0, 1, 0, 1]^T$ is an extreme event that we are designing against. Note that Y_1 and Y_3 are the randomized displacements of the masses m_1 and m_2, respectively. Y_2 and Y_4 are the corresponding randomized velocities. The equation component Y_5 can be interpreted as a force exerted on the suspension system by a maximum vehicle acceleration. The disturbances W_2, W_4, and W_5 are interpreted as additional random forces generated by operating over an uneven terrain.

Design objective. The vehicle design goal is to balance the support of the weight of the empty vehicle plus the vehicle load to minimize the stress on both the vehicle and the load. For the toy problem the design objective is interpreted as choosing masses m_1 and m_2 such that $m_1 + m_2 = 3$ (feasibility) and the energy dissipated through friction by each of the masses over the time interval $[0, T]$ are approximately the same (optimality). In particular, for time $T = 10secs$, let $b = [b_1, b_2]^T$ and $\mathcal{E}_1(b) = \int_0^{10} b_1 Y_2^2(t)\, dt$ and $\mathcal{E}_2(b) = \int_0^{10} b_2 Y_4^2(t)\, dt$ be the energies dissipated by friction, respectively. We want the mean and variance of $P(b) = \mathcal{E}_1(b) - \mathcal{E}_2(b)$ to be as close to zero as possible. The two uncertain friction

coefficients represent the uncertain operating environment. The measure of design performance $P(b)$ is stochastic with computable distributions depending on the friction coefficient values.

Designs. Feasible designs for the 3-spring system are determined by a single design parameter m_1 and the constraint $0 \leq m_1 \leq 3$. If $0 \leq m_1 \leq 3$ then the pair $(m_1, 3 - m_1)$ is a feasible design.

For additional realism, we assume that only a finite amount of data is available for the design problem. For $b_1 \in \{3/2, 7/4, \ldots, 4\}$, $b_2 \in \{1/4, 3/8, \ldots, 3/2\}$, and $m_1 \in \{1/2, 1, /, \ldots, 5/2\}$, the system was simulated 20,000 times over the interval $[0, 10]$ and the value of $P(b)$ recorded. There are five designs each operating in 121 different environments given by the pairs (b_1, b_2). The SDE model was used to generate the data but we could have started with five prototype designs and exhaustively tested the suspension systems in 121 different environments.

4.1.1 Efficient designs

For each (b_1, b_2) the difference of dissipated energy is a normal random variable. The assumption is based on an examination for the simulation data. Figure 3 depicts a histogram of $P(b_1, b_2, m_1)$ for $b_1 = 1.75$, $b_2 = 0.375$, and $m_1 = 1/2$. Let $\mu = E(P)$ and $\sigma^2 = var(P)$. Normality suggests using the decision surrogate $\mu + \sigma$, a function of (b_1, b_2, m_1), for choosing the "best" design, i.e., a design for which the risk or probability of $P > \mu + \sigma$ is approximately 0.15. For each pair (b_1, b_2) cubic splines are used to extend $\mu(b_1, b_2, \cdot) + \sigma(b_1, b_2, \cdot)$ to more values of m_1 in the interval $[0, 3]$ enlarging the design space. For $b_1 = 1.75$ and $b_2 = 0.375$ we have Figure 4 depicting the interpolating curve.

The decision surrogate for each design, for instance for $m_1 = 1.6328$ and $m_1 = 1.6719$, can be represented as a surface over the operating environments (see Figure 5).

The optimal design is sought minimizing $\mu + \sigma$ over all (b_1, b_2) . This is clearly impossible so we resort to multicriteria methods. Design $\bar{m}_1$ is said to dominate Design m_1 provided $\bar{\mu}(b_1, b_2) + \bar{\sigma}(b_1, b_2) \leq \mu(b_1, b_2) + \sigma(b_1, b_2)$ for all (b_1, b_2) with at least one strict inequality. We can concentrate on the set of efficient designs $\mathcal{D}$, i.e., Design m_1 is in $\mathcal{D}$ provided there is no Design $\bar{m}_1$ that dominates it. If Design m_1 is the only member of $\mathcal{D}$ then Design m_1 is preferred.

The literature has a number of preference rules for choosing a preferred efficient design (Ehrgott, 2005). One possible preference rule is to consider the lower envelope MS^* of the surfaces $\mu + \sigma$ for designs in $\mathcal{D}$. The surface MS^* is known as the ideal surface and is not the surrogate surface for any design. The design whose surrogate surface minimizes the distance to MS^* is the preferred design. Again, different metrics can be used, we use the ℓ_2 metric because of our concentration on inner product spaces.

In the example, the set of efficient designs contains 6 designs, with m_1 in the interval $[1.6328, 1.6719]$. The preferred design over all (b_1, b_2) is $m_1 = 1.6328$. The

distance of the surrogate surface for design $m_1 = 1.6328$ to the ideal surface is 0.0268. Notice that the design is preferred by the lower-level designer and higher-level information models play no role. We are left with the following question. Which design is preferred taking into account the interactions with the logistics design?

4.2   The portfolio problem

In contrast with the physics-based dynamic suspension model, the logistcis model is based on "soft" knowledge and intuition derived from experience with similar vehicles and missions. As with the suspension design we assume a late stage logistics design with many constraints imposed by earlier design stages including space and weight constraints. We will simplify the design problem for our purposes by assuming the problem is achieving an optimal partition of the available space divided between passengers and cargo.

The stochastic optimization problem with uncertainty becomes

$$\max U(t) = p^{3/2}U_1(t) + (1-p)^{2/3}U_2(t) \tag{2}$$

$$subject\ to\ 0.1 \le p \le 0.9,$$

where p is the design variable, $0 \le t \le 1$ represents the uncertain mission (normalized average trip length), U_1 is the random utility for passengers, and U_2 is the random utility for cargo. We assume for each t, that $U_i(t), i = 1, 2$, is a random variable with a triangular distribution density f_i and distribution $F_t^i(\cdot)$. We are suppressing, for the moment, the dependence of f_i and $F_t^i(\cdot)$ on t. The coefficients $(1-p)^{2/3}$ and $p^{3/2}$ were chosen as part of the numerical example. The effect of the weights is to emphasize U_1 for larger values of p. Mathematically, the optimization problem is nonlinear.

We are not attempting to model the decision maker with the stochastic utilities(see e.g., Blavatsky, 2008). Rather the model reflects an exogenous random influence of the situation on the objective usefulness of the passenger or cargo capacity. Thus the model assumption is about objective reality, not about the decision maker. The form of the functional $p^{3/2}U_1(t) + (1-p)^{2/3}U_2(t)$ results from designer preferences but this particular form was chosen for the illustrative example to provide reasonable results.

The density for a triangular distribution $F_t^i(\cdot)$ is given by three numbers $\{min, mode, max\}$. In our example, the distributions depend on the value of the uncertainty. Hence for each utility the three numbers are functions of the uncertainty t. In Figure 6, the densities $f_i(t)$ for the distributions $F_t^i(\cdot)$ of $U_i, i = 1, 2$, are given by the functions $\{min_i(t), mode_i(t), max_i(t)\}$ for $0 \le t \le 1$. Notice that $min_1(t) = min_2(t)$ and $max_1(t) = max_2(t)$. The two utilities differ in the most likely event for each, $mode_1(t)$ and $mode_2(t)$.

Without resolving the optimization problem we can estimate the value at risk (Rockafellar, 2007) at any confidence level $1 - \alpha$ for any design p and uncertainty

value t. For instance, see Figure 7. Clearly, the design $p = 4/5$ performs best for low average trip lengths and the design $p = 1/5$ for high average trip lengths. The designs $p = 2/5$ and $p = 3/5$ perform somewhere in between. The goal is a preferred design that performs "best" over the whole range of average trip lengths.

4.2.1 Conditional value at risk

In order to resolve the stochastic optimization problem (2) for a fixed value of the uncertainty we have to invoke a coherent measure of risk. Our choice for the coherent measure of risk is the "conditional value at risk" (Rockafellar and Uryasev, 2000, 2002) which enables us to use Rockafellar and Uryasev's remarkable result. Suppressing the dependence of the utility U on the uncertainty t and the logistics design p,

$$CVaR_\alpha(-U) = \min_{c \in R}\{c + (1-\alpha)^{-1}E(max\{0, \, -U - c\})\}. \tag{3}$$

The basic computation is $E(max\{0, \, -U - c\})$. For the fixed t, let

$$
\begin{aligned}
g_1(p, \, c) &= E(max\{0, \, -p^{3/2}U_1(t) - c\}) \\
&= \int_{-\infty}^{\infty} max\{0, \, -p^{3/2}s - c\}\, dF_t^1(s) \\
g_2(p, \, c) &= E(max\{0, \, -(1-p)^{2/3}U_2(t) - c\}) \\
&= \int_{-\infty}^{\infty} max\{0, \, -(1-p)^{2/3}s - c\}\, dF_t^2(s),
\end{aligned}
$$

where F_t^1 and F_t^2 are the distributions of $U_1(t)$ and $U_2(t)$, respectively, and $0.1 \le p \le 0.9$. Again, suppressing the dependence on the uncertainty t we obtain

$$CVaR_\alpha(-U) = \min_{c \in R}\{c + (1-\alpha)^{-1}(g_1(p, \, c) + g_2(p, \, c)\}.$$

The goal is the design p which minimizes $CVaR_\alpha(-U)$, for $0.1 \le p \le 0.9$. Thus we are left with the problem

$$minimize: \quad c + (1-\alpha)^{-1}(g_1(p, \, c) + g_2(p, \, c) \tag{4}$$

$$subject\ to: \quad 0.1 \le p \le 0.9 \ and \ c \in R,$$

which is easily solved using the MatLab function *fmincon*. The reader is cautioned not to optimize over $(t, \, p, \, c)$. Our approach requires a design p to be evaluated over all values of t, i.e., the uncertainty can assume any value in the interval $[0, \, 1]$.

4.2.2 The preferred logistics design

In the example, $\alpha = 0.85$. For each uncertainty value $t \in \{0, \, 1/64, \, \ldots, \, 1\}$, we resolve the corresponding stochastic optimization problem (4) for $-U$ using the conditional value at risk at confidence level α and the minimization rule (3). We

obtain sixty-five designs $\{p_j\}_{j=1}^{65}$ optimal for the corresponding uncertainty values $\{t_i\}_{i=1}^{65}$. For each of the optimal designs, we find the value at risk $VaR_{1-\alpha}(-U)$ at confidence level $1 - \alpha$, i.e., $VaR_{1-\alpha}(-U)$ is the solution of $F_{-U}(z) = 1 - \alpha$, as a function of t. Each design results in a 65-vector with components being the value at risk for particular values of the uncertainty. The goal is a design with the largest value at risk at confidence level $1 - \alpha$ for each uncertainty value.

The sixty-five optimization problems result in twelve different designs with p between 0.1 and 0.9, where each design is associated with a 65-vector of values at risk. Multicriteria methods are used to identify the set of efficient designs by comparing the vectors of values at risk.

We usually find that the set of efficient designs is a smaller (often much smaller) subset of the total set of designs. Unfortunately, in our example there is no reduction in the number of designs to be considered. The higher-level decision maker will consider how each of the twelve efficient logistic designs interacts with the efficient designs for the vehicle suspension system and choose the most compatible subdesigns for the vehicle design.

Of interest is a lower-level preferred efficient design. We find a preferred design by introducing an "ideal" point, for instance, the upper envelope of the value at risk vectors for the twelve designs, and by choosing the design, the preferred design, whose value at risk vector is closest to the ideal point measured by the ℓ_2-norm. In Figure 8, the preferred design $p = 0.1309$ is compared to the "all passenger" ($p = 0.9$) and "all cargo" ($p = 0.1$) designs that were additionally examined.

Again, notice that the lower-level design process makes no use of the higher-level information models. However, we still seek the design which is preferred taking into account the interactions with the vehicle suspension design. There is no reason that the subdesign preferred by the lower-level designer will be preferred by the higher-level designer concerned with tradeoffs between alternative lower-level subdiscipline designs.

The lower-level models and design methodologies are not part of the higher-level decision maker's knowledge base. The lower-level models and methods were chosen to illustrate in a small way the breadth of possibilities. Our paper is concerned with the higher-level models and the higher-level decision methodology. The higher-level models are to be information models using the second-order statistics of the performance functions of the lower-level designs.

4.3   The reduced-order component models

The construction of a reduced-order model for the utilization of the load capacity is simpler than for the simplified suspension system. Hence we will start with a reduced-order representation of random utility U of a partition of the load capacity as a function of the uncertain trip length.

4.3.1 Representation of the higher-level model of a logistics design

Suppose that the logistics design is fixed with utility function U. Let $\mu_B(t) = E(U(t))$ be the mean utility function estimated at the lower level and passed up to the next higher level. At the higher level, $\mu_B(t)$ is the proxy for the mean mission related maintenance. Note that $U(t) = \mu_B(t) + U(t) - E(U(t))$. Hence, in the spirit of the Wiener process simulation we are concerned with a higher-level representation of $V(t) = \int_0^t U(s) - E(U(s)) \, (ds)^{1/2}$, i.e., a linear transformation B with the property that $[BW_2](t)$ and $V(t)$ have the same covariance kernels, where W_2 is a Wiener process on $[0, 1]$.

Note that by assumption $E[U(s)U(t)] = E(U(s))E(U(t))$, and so $E[(U(s) - E(U(s)))(U(t) - E(U(t)))] = 0$ for $0 \le s, t \le 1$ and $s \ne t$. Following the pattern established for the Wiener process let $V(t) = \int_0^t (U(s) - E(U(s))) \, (ds)^{1/2}$, for $0 \le t \le 1$, with variance

$$
\begin{aligned}
k_3(t) &= E[(V(t))^2] \\
&= \frac{(p^3 + (1-p)^{4/3})(2t^3 - 3t^2 + 6t)}{108}.
\end{aligned}
$$

The variance of $V(t)$ is estimated at the lower level and passed up to the next higher level. At the higher level, the decision maker focuses on the increasing function k_3 defined on $[0, 1]$. For a given subinterval $[s, t]$ of $[0, 1]$, the ratio $(k_3(t) - k_3(s))/(t - s)$ is interpreted at the lower level as the average variance of the utility U on $[s, t]$. At the higher level, the ratio is the proxy for the average variance of mission related maintenance.

Let $R_3(s, t) = k_3(min(s, t))$ and BW_2 be the limit in distribution of $(R_{3tt}^C)^T Z_t$. Again, Z is a $(0, 1)$-normal function on $[0, 1]$. Since R_3 is the covariance kernel for V, $[BW_2](t)$ and $V(t)$ have the same covariance kernels and we conclude that B is a reduced-order representation of V. The discrete representation of B is $(R_{3tt}^C)^T$. The dependence of the reduced-order representation B of the centered logistics design p is illustrated by the plots in Figure 9.

4.3.2 Representation of a higher-level model of the balance of a suspension design

For a fixed Design m_1, we will show that analysis of the simulations supports

$$
E(P(x, y)P(\bar{x}, \bar{y})) = E(P(x, y))E(P(\bar{x}, \bar{y}))
$$

for $1.5 \le x, \bar{x} \le 4$, $0.25 \le y, \bar{y} \le 1.5$ and $(x, y) \ne (\bar{x}, \bar{y})$. Let $x_i = 1.5 + 0.25(i - 1)$ and $y_j = 0.25 +).125(j - 1)$, $i, j = 1, 2, \ldots, 11$. For random choices of (x_i, y_j) and (x_k, y_ℓ), $(x_i, y_j) \ne (x_k, y_\ell)$, we observe that $P(x_i, y_j) - E(P(x_i, y_j))$ and $P(x_k, y_\ell) - E(P(x_k, y_\ell))$ are uncorrelated. For instance, when $m_1 = 1/2$ the estimated correlation coefficient of $P(x_6, y_{10}) - E(P(x_6, y_{10}))$ and $P(x_{11}, y_{13}) - E(P(x_{11}, y_{13}))$ is -0.0050.

Because of the burden of a complete computation, we will rely on sampling from the 121^2 data points. Let

$$\bar{R}(x_i, y_j, x_k, y_\ell) = E[(P(x_i, y_j) - E(P(x_i, y_j)))(P(x_k, y_\ell) - E(P(x_k, y_\ell)))].$$

For a random sample $\mathcal{S}$ of 4-tuples (x_i, y_j, x_k, y_ℓ) of size 121, chosen without replacement and $(x_i, y_j) \neq (x_k, y_\ell)$, the sample mean of $\bar{R}(\mathcal{S})$ is 2.4220e-04 and the sample variance is 7.8929e-06.

Roughly, the same results hold for all five designs. We conclude that assuming $\bar{R}(x_i, y_j, x_k, y_\ell) = 0$ for $(x_i, y_j) \neq (x_k, y_\ell)$ is a reasonable modeling approximation for each design. Therefore the covariance kernel of $P - E(P)$ reduces to the variance field, a function of two variables, which determines $P - E(P)$.

In order to simplify the notation, we normalize the domain of the suspension balance P as $[0, 1] \times [0, 1]$ and introduce the transformed balance function $\bar{P}$. Let $\bar{P}(u, v) = P(1.5(1 - u) + 4.0u, 0.25(1 - v) + 1.5v)$ for $0 \leq u, v \leq 1$. Suppose that the suspension design is fixed with balance function $\bar{P}$. We are concerned with a representation of

$$\bar{Q}(u, v) = \int_0^u \int_0^v \bar{P}(x, y) - E(\bar{P}(x, y)) \, (dx)^{1/2} \, (dy)^{1/2},$$

i.e., a linear operator A with the property that AW_1 and $\bar{Q}$ have the same covariance kernels, where W_1 is a Wiener field on $[0, 1] \times [0, 1]$. Let $\mu_A(u, v) = E(\bar{P})$ be the mean balance function, a proxy at the higher level of expected maintenance resulting from the operating environment.

For the variance $\sigma_A^2(u, v)$ of the higher-level balance function $\bar{Q}(u, v)$,

$$\begin{aligned} \sigma_A^2(u, v) \\ &= E[\bar{Q}^2(u, v)] \\ &= \int_0^u \int_0^v E[(\bar{P}(x, y) - E(\bar{P}(x, y)))^2] \, dy \, dx. \end{aligned}$$

Because $\bar{Q}$ is a function of two variables, we introduce two increasing functions $k_1(u) = \sigma_A^2(u, 1)/\sigma_A(1, 1)$ and $k_2(v) = \sigma_A^2(1, v)/\sigma_A(1, 1)$. Let $R_1(u, \bar{u}) = k_1(min(u, \bar{u}))$ and $R_2(v, \bar{v}) = k_2(min(v, \bar{v}))$. Notice that R_1 and R_2 are defined in terms of the marginal distributions of $\bar{Q}$.

Let R_{1uu}^C be an upper triangular matrix such that $(R_{1uu}^C)^T \cdot R_{1uu}^C = R_{1uu}$. Define R_{2vv}^C in a similar way. We define AW_1 to be the limit in distribution of $(R_{1uu}^C)^T Z_{uv} R_{2vv}^C$, where Z is a $(0, 1)$-normal function on $[0, 1] \times [0, 1]$. For the numerical example, the analysis of the simulations supports that the covariance of AW_1 is a good approximation to the covariance of $\bar{Q}$.

The lower-level designers pass up to the next level for each design the expected performances of the suspension design $\mu_A(u, v) = E(P(u, v))$ and of the logistic design $\mu_B(t) = E(U(t))$. In addition, the two operators A and B are passed up in the form of the increasing functions $k_1(u)$, $k_2(v)$, and $k_3(t)$ that determine the operators. In the next section, the reduced-order information model of each vehicle design will be developed in terms of μ_A, μ_B, k_1, k_2, and k_3.

5 Construction of the higher-level decision model

In this section, we assume the reduced-order component models in the transformation sense and matrix realizations of approximate reduced-order component models using the lower-level estimates of the second-order statistics of the design performance fields. The matrix realizations are used to produce simulations of higher-level information models of the two-component system illustrated in Figure 1. In Section 5.2.4 , generic formulas are given for constructing more complex system models from reduced-order component models with more than two components.

The section concludes with multicriteria decision methods employed to produce a preferred multicomponent design using data from the vehicle model. The data could be taken as results of a prototype testing program. The presentation is illustrated with the simplified example but the methodology is general and can be applied to larger, more realistic problems.

5.1 Interactions of the two components

Folklore has it that in conflicts between subdesigns the model with the most physics wins. If this held for our toy problem then the suspension system designer would be able to dictate to the logistics designer slighting mission concerns. However, mission concerns pay the bills. Can the higher-level decision maker reach a reasonable compromise producing a vehicle that is both useful and maintainable? For a "reasonable compromise" the tradeoffs should be quantifiable, the preference rule should be explicit or archivable, and the decision should guide lower-level designers to improvements in later stage designs. "Just politics" is not an adequate lower-level interpretation of higher-level decisions.

A design for the suspension system and a logistics design for the load capacity interact as follows. Expected vehicle and load stress is given for each suspension design. Also, expected vehicle utility is given for each payload design. Notice that the covariance kernel of AW_1 is the covariance kernel of $\bar{Q}$. Similarly, V and BW_2 have the same covariance kernels. Assume that $\mu_A + AW_1$ is an acceptable reduced-order approximation for the performance field for the suspension system performance and $\mu_B + BW_2$ is an acceptable reduced-order approximation for the performance process for the payload design. Further, assume that the influences of $\bar{Q}$ on V and of V on $\bar{Q}$ do not affect the expected performance of either, only the respective variances that are related to risk.

We assume that increased payload utility results in increased suspension imbalance, but increased suspension imbalance results in decreased payload utility. Thus, we have conflicting component design goals: maximize payload utility and minimize suspension imbalance. Formally, assume (see Figure 1) that

$$\bar{Q} = G_1 = A(aW_1 + bG_2),$$

where a, $b \geq 0$ and $a + b = 1$, and

$$V = G_2 = B(cW_2 - dG_1),$$

where c, $d \geq 0$ and $c + d = 1$.

Thus the performance variance (or performance variability) of the suspension design is the result of applying A to the exogenous disturbance $aW_1 + bG_2$. Notice that we are replacing a portion of the uncolored disturbance with a "colored" disturbance, i.e., bG_2 is the "colored" portion of the background disturbance affecting $\bar{Q}$. The technical details, including extensions of A and B, will be provided in the next subsection. Similarly, the performance variance (or performance variability) of the load capacity design is the result of applying B to the exogenous disturbance $cW_2 - dG_1$. Concurrent design requires a higher-level decision maker to coordinate tradeoffs between the lower-level designs in the presence of uncertainty and risk.

5.2 Matrix realizations of the approximate reduced-order models

The matrix realizations of the discretized models provide the basis for numerical models and implementation of a decision methodology. At this point, considering the reduced- order models more abstractly is useful. The presentation, already complicated, is simplified by ignoring the physical and and probabilistic interpretations of the models. Further, the results are general in the sense that other design coordination problems can be considered using the formulas developed in this section.

5.2.1 Extensions of the higher level models A and B

We extend A to functions of three variables as follows:

$$[(AH)_{uvt}](\ell, m, n)$$
$$= [(R_{1uu}^C)^T (K_{uu}^C)^{-T} H_{uvt}(\cdot, \cdot, n)(K_{vv}^C)^{-1} R_{2vv}^C](\ell, m).$$

In order to simplify notation, let $A_\kappa = R_{1uu}^C$ and $A_\nu = R_{2vv}^C$. Then

$$[(AH)_{uvt}](\ell, m, n) = [A_\kappa^T (K_{uu}^C)^{-T} H_{uvt}(\cdot, \cdot, n)(K_{vv}^C)^{-1} A_\nu](\ell, m).$$

In particular, when $H = W$, we have

$$[(AW)_{uvt}](\ell, m, n)$$
$$= \sum_{k=1}^{n} [(K_{tt}^C)^T](n, k))[A_\kappa^T](\ell, \cdot)Z_{uvt}(\cdot, \cdot, k)A_\nu(\cdot, m). \tag{5}$$

Notice the close relationship of Equations (1) and (5).

If we choose $A_\lambda = K_{tt}^C$ then

$$[(AW)_{uvt}](\ell, m, n)$$
$$= \sum_{i=1}^{\ell}\sum_{j=1}^{m}\sum_{k=1}^{n}[A_\kappa^T](\ell, i)[A_\nu^T](m, j)[A_\lambda^T](n, k)Z_{uvt}(i, j, k).$$

Extend B to functions of three variables as follows:

$$[(BH)_{uvt}](\ell, m, n) = [B_\lambda^T(K_{tt}^C)^{-T}H_{uvt}](\ell, m, \cdot)](n).$$

Also, if we choose $B_\kappa = K_{uu}^C$ and $B_\nu = K_{vv}^C$, then

$$[(BW)_{uvt}](\ell, m, n)$$
$$= \sum_{i=1}^{\ell}\sum_{j=1}^{m}\sum_{k=1}^{n}[B_\lambda^T](\ell, i)[B_\nu^T](m, j)[B_\lambda^T](n, k)Z_{uvt}(i, j, k).$$

Therefore

$$[(ABH)_{uvt}](\ell, m, n)$$
$$= \sum_{k=1}^{n}[B_\lambda^T(K_{tt}^C)^{-T}](n, k)[A_\kappa^T(K_{uu}^C)^{-T}H_{uvt}(\cdot, \cdot, k)(K_{vv}^C)^{-1}A_\nu](\ell, m)$$

and

$$[(ABW)_{uvt}](\ell, m, n)$$
$$= \sum_{i=1}^{\ell}\sum_{j=1}^{m}\sum_{k=1}^{n}[A_\kappa^T](\ell, i)[A_\nu^T](m, j)[B_\lambda^T](n, k)Z_{uvt}(i, j, k). \tag{6}$$

Similarly

$$[(BAH)_{uvt}](\ell, m, n)$$
$$= \sum_{k=1}^{n}[B_\lambda^T(K_{tt}^C)^{-T}](n, k)[A_\kappa^T(K_{uu}^C)^{-T}H_{uvt}(\cdot, \cdot, k)(K_{vv}^C)^{-1}A_\nu](\ell, m) \tag{7}$$
$$= [(ABH)_{uvt}](\ell, m, n).$$

5.2.2 Separable approximations

Referring to Figure 1, the higher-level design decisions are based on the interaction functions G_1 and G_2.

One would not expect that $(A + B)^{1/2} = A^{1/2} + B^{1/2}$ or that the sum of two separable fields is separable (refer to the comments on separable fields in Section

2). Separable approximations of $I + AB$, discussed in Reneke and Samson (2008), play a role at the higher level in the construction of multiple-component system models. We will continue the discussion of these technical matters in the context of our example in the next section. To repeat, a complete understanding of the details is not necessary to apply the modeling methods, i.e., the designer can proceed in terms of the finite discrete approximations.

Given the interaction functions G_1 and G_2,

$$
\begin{aligned}
G_1 &= A(aW_1 + bG_2) \\
G_2 &= B(cW_2 - dG_1),
\end{aligned}
$$

we proceed formally omitting necessary details. We express G_1 and G_2 in terms of A and B as

$$
\begin{aligned}
G_1 &= (I + bdAB)^{-1}(aAW_1 + bcABW_2) & (8) \\
G_2 &= (I + bdBA)^{-1}(cBW_2 - adBAW_1). & (9)
\end{aligned}
$$

Note that in (6) and (7) we have already shown how to compute ABW and BAW and now turn to the existence of inverses $(I + bdAB)^{-1}$ and $(I + bdBA)^{-1}$. Each inverse has the form of a desired operator arrived at through formal manipulations but is ultimately meaningless. Also, since the fields $W + bdABW$ and $W + bdBAW$ are not separable, the inverses are difficult to compute. However, we can approximate the desired operator by replacing $I + bdAB$ with a separable approximation and attach a meaning to $(I + bdAB)^{-1}$.

We turn to separable approximations with analytic inverses. To simplify the formulas, let $\hat{Y}_{1uvt} = (W + bdABW)_{uvt}$ and $\hat{Y}_{2uvt} = (W + bdBAW)_{uvt}$. The approximations require the computation of both $var(\hat{Y}_1(u_\ell, v_m, t_n))$ and $var(\hat{Y}_2(u_\ell, v_m, t_n))$. We see that

$$
\begin{aligned}
&var(\hat{Y}_1(u_\ell, v_m, t_n)) \\
&= u_\ell v_m t_n + 2bd[A_\kappa^T K_{uu}^C](\ell, \ell)[(K_{vv}^C)^T A_\nu](m, m)[B_\lambda^T K_{tt}^C](n, n) \\
&\quad + b^2 d^2 k_1(u_\ell) k_2(v_m) k_3(t_n).
\end{aligned}
$$

A separable approximation of $\hat{Y}_{1uvt}$ requires three matrices which, for the moment, we will denote by C_κ, C_λ, and C_ν. We require that these matrices satisfy the following conditions:

$$
\begin{aligned}
var(\hat{Y}_1(u_\ell, v_M, t_N)) &= [C_\kappa^T C_\kappa](\ell, \ell)[C_\nu^T C_\nu](M, M)[C_\lambda^T C_\lambda](N, N) & (10) \\
var(\hat{Y}_1(u_L, v_m, t_N)) &= [C_\kappa^T C_\kappa](L, L)[C_\nu^T C_\nu](m, m)[C_\lambda^T C_\lambda](N, N) & (11) \\
var(\hat{Y}_1(u_L, v_M, t_n)) &= [C_\kappa^T C_\kappa](L, L)[C_\nu^T C_\nu](M, M)[C_\lambda^T C_\lambda](n, n). & (12)
\end{aligned}
$$

We can obtain the matrices as follows. Let

$$
\begin{aligned}
k_4(u_\ell) &= var(\hat{Y}_{1uvt}(\ell, M, N))/var(\hat{Y}_{1uvt}(L, M, N))^{2/3} \\
k_5(v_m) &= var(\hat{Y}_{1uvt}(L, m, N))/var(\hat{Y}_{1uvt}(L, M, N))^{2/3} \\
k_6(t_n) &= var(\hat{Y}_{1uvt}(L, M, n))/var(\hat{Y}_{1uvt}(L, M, N))^{2/3}.
\end{aligned}
$$

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Note that $k_4(1) = k_5(1) = k_6(1) = var(\hat{Y}_{1uvt}(L, M, N))^{1/3}$. As we have done before, let

$$R_4(u, \bar{u}) = k_4(min(u, \bar{u})) \qquad (13)$$
$$R_5(v, \bar{v}) = k_5(min(v, \bar{v})) \qquad (14)$$
$$R_6(t, \bar{t}) = k_6(min(t, \bar{t})) \qquad (15)$$

and define

$$C_\kappa = R_{4uu}^C \qquad (16)$$
$$C_\nu = R_{5vv}^C \qquad (17)$$
$$C_\lambda = R_{6tt}^C. \qquad (18)$$

Using (16-18) and (13-15), we calculate

$$[C_\kappa^T C_\kappa](\ell, \ell)[C_\nu^T C_\nu](M, M)[C_\lambda^T C_\lambda](N, N)$$
$$= R_4(u_\ell, u_\ell)R_5(1, 1)R_6(1, 1)$$
$$= k_4(u_\ell)k_5(1)k_6(1)$$
$$= var(\hat{Y}_1(u_\ell, v_M, t_N)),$$

which shows that matrices C_κ, C_λ and C_ν defined in (16-18) satisfy condition (10). Similarly, these matrices satisfy conditions (11) and (12).

Let C be the operator defined by

$$[CW](u_\ell, v_m, t_n) = \sum_{i=1}^{\ell} \sum_{j=1}^{m} \sum_{k=1}^{n} C_\kappa^T(\ell, i)C_\nu^T(m, j)C_\lambda(n, k)Z_{uvt}(i, j, k).$$

CW is the separable approximation of $\hat{Y}_{1uvt} = (W + bdABW)_{uvt}$, as shown in Reneke and Samson (2008). The separable approximation of $\hat{Y}_{2uvt} = (W + bdBAW)_{uvt}$ is constructed in the same manner.

5.2.3 Separable approximations for $(I + bdAB)^{-1}$

Let C be the separable approximation of the field $I + bdAB$ and C^{-1} be the separable operator defined by

$$(C^{-1})_\kappa = K_{uu}^C(C_\kappa)^{-1}K_{uu}^C$$
$$(C^{-1})_\nu = K_{vv}^C(C_\nu)^{-1}K_{vv}^C$$
$$(C^{-1})_\lambda = K_{tt}^C(C_\lambda)^{-1}K_{tt}^C.$$

Note that $(C^{-1})_\kappa(K_{uu}^C)^{-1}C_\kappa(K_{uu}^C)^{-1} = K_{uu}^C(C_\kappa)^{-1}K_{uu}^C(K_{uu}^C)^{-1}C_\kappa(K_{uu}^C)^{-1} = I$
Similarly, $C_\nu(K_{vv}^C)^{-1}(C^{-1})_\nu(K_{vv}^C)^{-1} = I$ and $(C^{-1})_\lambda(K_{tt}^C)^{-1}C_\lambda(K_{tt}^C)^{-1} = I$.

If H_1 is a Hellinger integrable function with $H_1(0, v, t) = H_1(u, 0, t) = H_1(u, v, t) = 0$ and $H_2 = CH_1$ then

$[C^{-1}H_2](u_\ell, v_m, t_n)$

$\begin{aligned}
&= [(C^{-1})_\kappa^T (K_{uu}^C)^{-T}(\sum_{k=1}^{n}[(C^{-1})_\lambda^T(K_{tt}^C)^{-T}](n, k)H_{2uvt}(\cdot, \cdot, k))(K_{vv}^C)^{-1}(C^{-1})_\nu](\ell, m) \\
&= \sum_{k=1}^{n}\sum_{\bar{k}=1}^{n}[(C^{-1})_\lambda^T(K_{tt}^C)^{-T}](n, k)[C_\lambda^T(K_{tt}^C)^{-T}](k, \bar{k})H_{1uvt}(\ell, m, \bar{k})) \\
&= \sum_{\bar{k}=1}^{n} I_{tt}(n, \bar{k})H_{1uvt}(\ell, m, \bar{k}) \\
&= H_{1uvt}(\ell, m, n)
\end{aligned}$

and so C^{-1} is the inverse of the separable approximation C of $I + bdAB$. Based on this derivation, we have

$$G_1 \sim C^{-1}(aAW_1 + bcABW_2) \tag{19}$$
$$G_2 \sim C^{-1}(cBW_2 - adBAW_1). \tag{20}$$

5.2.4 The algebra of discrete operators

From here to the end of the section we are not concerned with the origin of the reduced order representations of the component models but rather with the construction of information models of the system and the use of information models in decision making. We can build more complex information models by observing the following rules. Assuming that A and B are known separable nonnegative operators on the space of Hellinger integrable functions on $[0, 1] \times [0, 1] \times [0, 1]$ we have

$$\begin{aligned}
(AB)_\kappa &= B_\kappa(K_{uu}^C)^{-1}A_\kappa & (21) \\
(AB)_\nu &= B_\nu(K_{vv}^C)^{-1}A_\nu & (22) \\
(AB)_\lambda &= B_\lambda(K_{tt}^C)^{-1}A_\lambda & (23) \\
(A^{-1})_\kappa &= K_{uu}^C(A_\kappa)^{-1}K_{uu}^C & (24) \\
(A^{-1})_\nu &= K_{vv}^C(A_\nu)^{-1}K_{vv}^C & (25) \\
(A^{-1})_\lambda &= K_{tt}^C(A_\lambda)^{-1}K_{tt}^C & (26) \\
k_4(u) &= var([W + AW](u, 1, 1)) & (27) \\
k_5(v) &= var([W + AW](1, v, 1)) & (28) \\
k_6(t) &= var([W + AW](1, 1, t)) & (29) \\
R_4(u, \bar{u}) &= k_4(min(u, \bar{u})) & (30) \\
R_5(v, \bar{v}) &= k_5(min(v, \bar{v})) & (31) \\
R_6(t, \bar{t}) &= k_6(min(t, \bar{t})) & (32) \\
(I + A)_\kappa &\sim R_{4uu}^C & (33)
\end{aligned}$$

$$(I + A)_\nu \sim R^C_{5vv} \tag{34}$$

$$(I + A)_\lambda \sim R^C_{6tt}. \tag{35}$$

The notation A, B, etc. is meant to be generic, i.e., not referencing the specific operators, etc. of the illustrative example. Notice that equations (27-29) would have to be expanded. The fifteen formulas can serve as the basis for numerical simulations, an easy task using MatLab.

5.3   Simulation of the higher-level information model

From (19), the simulation of G_1 becomes

$$G_1(u_\ell,\, v_m,\, t_n) = a[(C^{-1}AW_1)_{uvt}](\ell,\, m,\, n) + bc[C^{-1}ABW_2)_{uvt}](\ell,\, m,\, n)$$

and so

$$
\begin{aligned}
&var(G_1(u_\ell,\, v_m,\, t_n)) \\
&= a^2[(C^{-1})^T_\kappa (K^C_{uu})^{-1} R_1 (K^C_{uu})^{-1}(C^{-1})_\kappa](\ell,\, \ell) \cdot \\
&\quad [(C^{-1})^T_\nu (K^C_{vv})^{-1} R_2 (K^C_{vv})^{-1}(C^{-1})_\nu](m,\, m) \cdot k_6(t_n) \\
&\quad + b^2 c^2 k_4(u_\ell) k_5(v_m)[(C^{-1})^T_\lambda (K^C_{tt})^{-1} R_3 (K^C_{tt})^{-1}(C^{-1})_\lambda](n,\, n).
\end{aligned}
$$

We are lucky in that $var(\hat{Y}_2(\ell,\, m,\, n)) = var(\hat{Y}_1(\ell,\, m,\, n))$. Therefore, the simulation of G_2 becomes

$$G_2(u_\ell,\, v_m,\, t_n) = c[(C^{-1}BW_2)_{uvt}](\ell,\, m,\, n) - ad[C^{-1}BAW_1)_{uvt}](\ell,\, m,\, n)$$

and

$$
\begin{aligned}
&var(G_2(u_\ell,\, v_m,\, t_n)) \\
&= a^2 d^2[(C^{-1})^T_\kappa (K^C_{uu})^{-1} R_1 (K^C_{uu})^{-1}(C^{-1})_\kappa](\ell,\, \ell) \cdot \\
&\quad [(C^{-1})^T_\nu (K^C_{vv})^{-1} R_2 (K^C_{vv})^{-1}(C^{-1})_\nu](m,\, m) \cdot k_6(t_n) \\
&\quad + c^2 k_4(u_\ell) k_5(v_m)[(C^{-1})^T_\lambda (K^C_{tt})^{-1} R_3 (K^C_{tt})^{-1}(C^{-1})_\lambda](n,\, n).
\end{aligned}
$$

The computations are easy because of our deliberate choices $A_\lambda = K^C_{tt}$, $B_\kappa = K^C_{uu}$, and $B_\nu = K^C_{vv}$. Other choices would introduce more realism but complicate the computations.

The following special cases are easy to check. If $bd = 0$ then $C^{-1} = I$.

If $b = 0$ then $G_1 = AW_1$ and $var(G_1(u_\ell,\, v_m,\, t_n)) = k_1(u_\ell)k_2(v_m)$. Also, $G_2 = cBW_2 - dAW_1$ and $var(G_2(u_\ell,\, v_m,\, t_n)) = c^2 k_3(t_n) + d^2 k_1(u_\ell)k_2(v_m)$.

If $d = 0$ then $G_1 = AW_1 + bBW_2$ and $var(G_1(u_\ell,\, v_m,\, t_n)) = a^2 k_1(u_\ell)k_2(v_m) + b^2 k_3(t_n)$. Also, $G_2 = BW_2$ and $var(G_2(u_\ell,\, v_m,\, t_n)) = k_3(t_n)$.

The decision surrogates. Of course, the operators A and B in the example depend on the designs of the suspension and payload partition, respectively. We have chosen to simplify our notation by not including this dependence explicitly in the notation.

Suppose that M_1 is a deterministic surrogate for "imbalance", i.e., in the absence of exogenous random disturbances including the influence of other components, the design minimizing M_1 should be preferred by the higher-level decision maker. Possibilities for M_1, a function of the uncertainties determined by the "suspension design", are dependent on the application. Thus M_1 is information the higher-level decision maker can use in coordinating various subcomponent designs. M_1 is provided by the lower-level designer and so approved at the lower level for use in "determining maintenance requirements". The lower-level designer is saying that the performances of his/her designs are properly characterized for the higher-level use by M_1. Everything depends on this understanding. Similar remarks hold for M_2 as a surrogate for "usefulness and the logistics planner".

The higher-level two-component decision surrogate becomes

$$MS_1(\ell,\, m,\, n) = M_1(\ell,\, m) + sqrt(var(G_1(\ell,\, m,\, n))) \tag{36}$$

$$MS_2(\ell,\, m,\, n) = M_2(n) - sqrt(var(G_2(\ell,\, m,\, n))). \tag{37}$$

A system Design α is said to dominate a system Design β if, introducing an explicit dependence on the design, $MS_1(\ell,\, m,\, n,\, \alpha) \leq MS_1(\ell,\, m,\, n,\, \beta)$ and $MS_2(\ell,\, m,\, n,\, \alpha) \geq MS_2(\ell,\, m,\, n,\, \beta)$ for all $(\ell,\, m,\, n)$ with at least one strict inequality. Again, a preference rule will have to be applied in case there is no single higher-level design dominating all others.

5.4   Tradeoff decisions for the complete vehicle model

Recall that G_1 is the performance of a suspension design coupled with a a logistics design. G_2 is the performance of a logistics design coupled with a suspension design. There are six suspension designs and twelve logistics designs forming seventy-two composite designs at the higher level. Each composite design is evaluated in four operating environments and for seven missions chosen to illustrate the method. The uncertain parameter pairs $(b_1,\, b_2)$ (operating environments) are $\{(2.2143, 0.6071), (2.2143, 1.2024), (3.4048, 0.6071), (3.4048, 1.2024)\}$. The normalized trip lengths (missions) are $\{0.0781, 0.2031, 0.3281\ 0.4531, 0.5781, 0.7031, 0.8281\}$. Obviously, the require data for the decision problem grows combinatorially which justifies the interest in reduced-order component models and vehicle models.

Modeling the level of component interactions, i.e., choosing b and d, determines the preferred component designs. We assume that either the higher-level decision maker has some additional knowledge or intuition or can explore the consequences of various combinations of b and d until a satisfactory design is obtained. Our position is that models do not make decisions but are an aid for the decision maker concentrating attention on the essential elements of the physical artifact.

Below we discuss possible choices of b and d affecting the interaction the two components. For each choice, the seventy-two designs were evaluated using the domination rule (36-37) and the ℓ_2 metric was employed to identify a preferred design.

Recall that $a = 1 - b$ and $c = 1 - d$. For instance, if $b = 0$ then $G_1 = AW_1$ and the higher-level preferred suspension design is $m_1 = 1.6406$. The lower-level preferred suspension design from Section 4.1.1 is $m_1 = 1.6328$. Further, $var(G_1(u_\ell, v_m, t_n)) = k_1(u_\ell)k_2(v_m)$, $G_2 = cBW_2 - dAW_1$, and $var(G_2(u_\ell, v_m, t_n)) = c^2 k_3(t_n) + d^2 k_1(u_\ell)k_2(v_m)$.

If $d = 0$ then $G_1 = AW_1 + bBW_2$, $var(G_1(u_\ell, v_m, t_n)) = a^2 k_1(u_\ell)k_2(v_m) + b^2 k_3(t_n)$, $G_2 = BW_2$, and the higher-level preferred logistics design is $p = 0.4279$. The lower-level preferred logistics design from Section 4.2.2 is $p = 0.1309$. Also $k_3(t_n) = var(G_2(u_\ell, v_m, t_n))$. The discrepancy can be attributed to two factors. The composite design is based on a higher-level model and the coupling resulting from multicriteria methods.

If $b = d = 1$ then the higher-level preferred vehicle design is $m_1 = 1.6797$ and $p = 0.4279$. If $b = d = 1/2$ then the higher-level preferred vehicle design is $m_1 = 1.6328$ and $p = 0.3580$. In the latter case, some surrogates for G_1, $t = 0.0781$, 0.4531, 0.8281, and G_2, $(b_1, b_2) = (2.2143, 0.6071)$, $(2.2143, 1.2024)$, $(3.4048, 0.6071)$, $(3.4048, 1.2024)$, are plotted in Figure 10.

The variety of possible preferred vehicle designs using different modeling assumptions are listed the following table.

b	d	m_1	p
0	0	1.6406	0.4279
1/2	0	1.6406	0.4279
1	0	1.6797	0.4279
0	1/2	1.6406	0.4279
1/2	1/2	1.6328	0.3580
1	1/2	1.6797	0.4279
0	1	1.6406	0.4279
1/2	1	1.6328	0.3085
1	1	1.6797	0.4279

Archiving the results. A potential benefit of the design process divided into stages is the availability of the design history to designers in the current stage, clarifying goals and justifying constraints. We would like to be able to review a decision at a later time, perhaps changing the preference rule, the confidence level, or the interaction parameters. What is needed in a particular case?

- Exploring alternate preference rules requires the decision variables for each subdesign.

- Exploring alternate confidence levels requires the second-order statistics of G_1 and G_2 for each subdesign.

- Exploring alternate interaction parameters (a, b, c, d) requires the representations of all of the operators for each subdesign.

We are not advocating changing the design decisions of a previous stage but, rather, maintaining the spirit of a previous stage in a long design process.

5.5   Section summary

Motivated by the example, we have outlined a general approach to developing an information model with two components and three uncertainties. The key step is producing the reduced-order models AW and BW. The variety of possible component models prevents us from laying out ground rules for this step. We have illustrated the fundamentals for two quite different component models.

Based on the system model, tradeoffs can be made by the higher-level decision maker using multicriteria methods to produce a preferred system design. Because there is no single way of accomplishing this goal, we have illustrated one way in the example.

Using obvious extensions of the operator algebra, the methods can be applied to decision problems with more components and more uncertainties. While problems can grow in size and complexity rapidly, the design stage/design level paradigm is intended to limit the growth. We have run test problems with three components and two uncertainties.

6   Concluding remarks

We have proposed an information model approach to the design of complex (multicomponent) engineering systems which is based on the algebra of reduced-order representations. Higher-level design decisions, the result of tradeoffs between alternative component designs, are made in terms of the second-order statistics of the component response fields. Since the statistics are functions of the uncertainties, multicriteria optimization methods are used to determine preferred higher-level designs.

Modeling the uncertainties as independent variables changes our viewpoint freeing up possibilities. In particular, shifting from time to the uncertainties as the independent variables tying component models together eliminates the time-scale problem. The higher-level coordination of lower-level designs eliminates the curse of dimensions and the problem of incompatible mathematical representations. Our use of the Central Limit Theorem in the construction of the higher- level decision model simplifies the higher-level decision making by moving all of the stochastic modeling into the Gaussian framework. The introduction of reduced-order representations and separable approximations support an algebra of linear operators as a modeling tool available to a wide class of decision makers. Multicriteria optimization framework allows lower-level designers to make use of tools from the measures of risk literature in the presence of Knight's uncertainty. We conclude that an

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information model approach to multidisciplinary design optimization is feasible for complex systems with incompatible component models.

We have chosen the illustrative example for the variety of possibilities provided: multi-scale time and length, dynamic models paired with static models, data based analysis paired with analysis based on expert opinion, multiple knowledge bases, different attitudes toward risk, etc.

In this exposition, we have presented one stage of the design process and illustrated the approach assuming two levels and two components. We have not addressed other issues of the overall design process such as multiple stages, levels, and components, which will naturally motivate future research directions.

Assuming multiple design stages, in the next stage the lower-level designers will know which of their efficient designs was preferred. Using this information as a starting point, the inference is that some subrange of the uncertainties is less important so that in the next stage, the previous-stage range will be reduced. The higher-level designer will have a better understanding of possible component designs. Goals can be adjusted to fit the achievable possibilities.

For complex systems at advanced design stages, we would expect several decision levels. Each lower level would pass up all efficient coordinated designs, i.e., the second-order statistics of the performance of each alternative. At each level, the decision process would reduce the set of feasible designs but the numbers passed up would grow emphasizing the importance of analytic methods and reduced-order models. There would be no preferred design until the final top level.

Our concern in developing a design philosophy for large complex systems has been preserving a role for engineering judgment at each level. Transparency of the approach is easier to maintain when the number of components in the coordination decision is small. A reasonable maximum for the number of components might be three. Remember that the higher-level decision maker might not understand the lower-level models or design methods. With our approach we intend to assist the designer rather than usurp his or her proper role and believe that models and methods should be a tool for a design decision.

As the design focus passes through levels, up and down, uncertainties may be added or dropped during the process. Note that higher-level designers will face uncertainties which are meaningless to lower-level designers. As we have already mentioned, manufacturing costs might be inappropriate as a factor for lower-level designers. Similarly, lower-level designers will face uncertainties which are meaningless to higher-level designers. For instance, geometric constraints limiting the number of passengers with their mission mandated equipment might be meaningless to the higher- level designer. While we have only considered a few simple possibilities, schemes for adding and dropping uncertainties should be explored.

While not discussed in this paper, our approach based on multicriteria methods satisfy Savage's first four postulates guaranteeing rational decisions. This is important in light of Hazelrigg's powerful criticism of current design selection methods Hazelrigg (2003). Further, the use of multicriteria methods is an important consideration for decisions in the presence of uncertainty as illustrated by the discussion

of Ellsberg's famous urn paradox elsewhere (Samson and Reneke, 2009).

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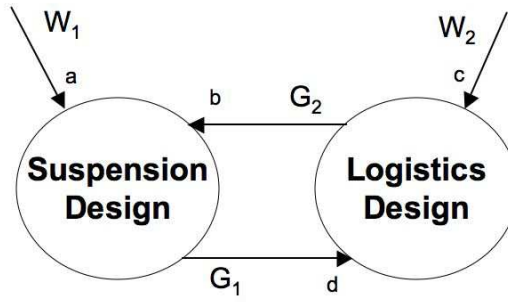
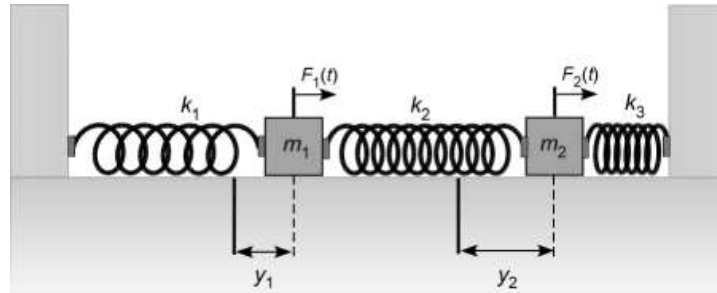Figure 1. The conceptual model of the the interactions of the suspension and logistic designs.
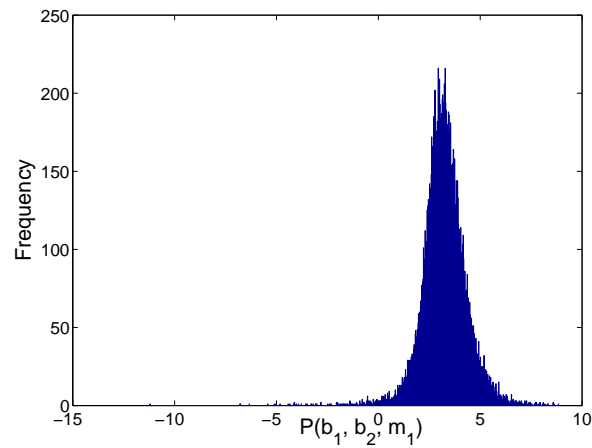

Figure 2. The 3-spring system.

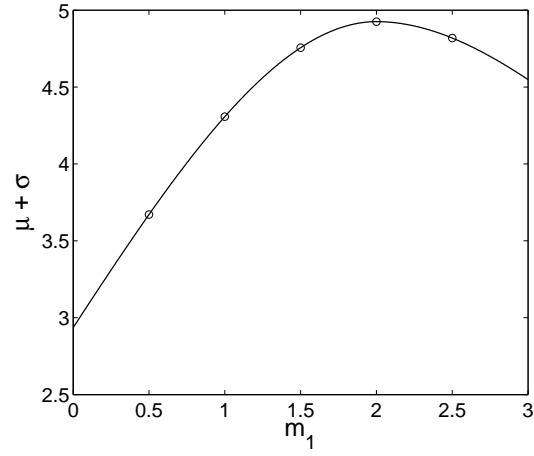Figure 3. Difference of dissipated energy for $b_1 = 1.75$, $b_2 = 0.375$, and $m_1 = 1/2$.

Figure 4. The interpolating curve of $\mu(b_1, b_2, \cdot) + \sigma(b_1, b_2, \cdot)$ for $b_1 = 1.75$ and $b_2 = 0.375$.
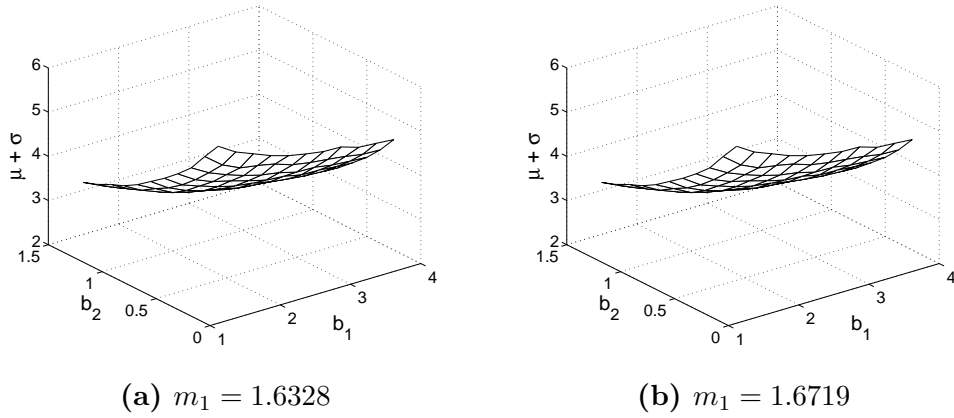

(a) $m_1 = 1.6328$ **(b)** $m_1 = 1.6719$

Figure 5. Surfaces $\mu(\cdot, \cdot, m_1) + \sigma(\cdot, \cdot, m_1)$.

(a) U_1 **(b)** U_2
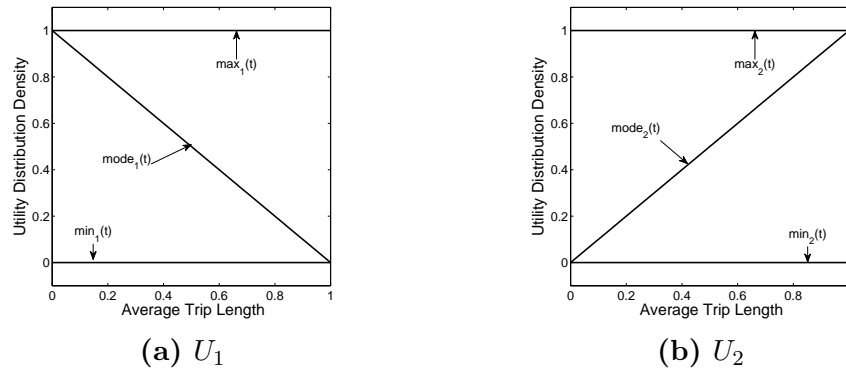
Figure 6. The functions $\{min_i(t), mode_i(t), max_i(t)\}$ for $0 \le t \le 1$ for the distributions of U_1 and U_2.
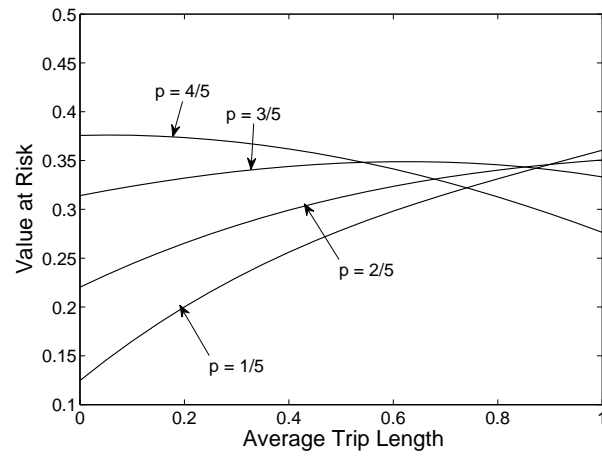
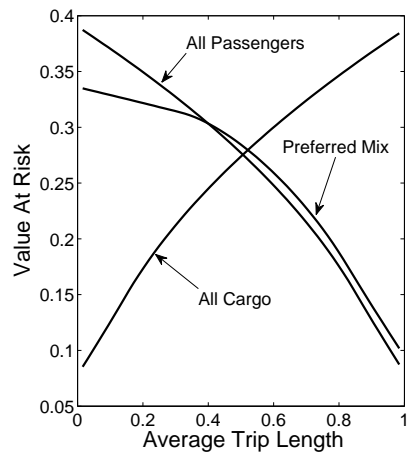Figure 7. Value at risk for four designs with $\alpha = 0.85$.

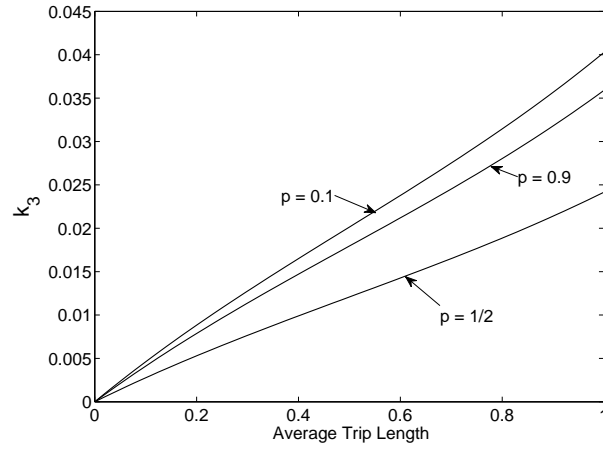Figure 8. The preferred design compared with the "all cargo" design and the 'all passengers" design.

Figure 9. Plots of k_3 defining the reduced-order representations of B for three different logistics designs.
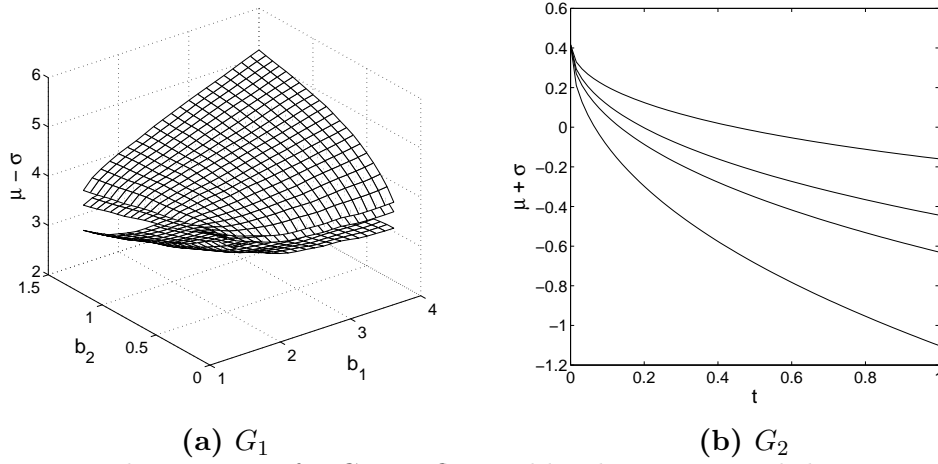

(a) G_1

(b) G_2

Figure 10. The surrogate for G_1 is influenced by the mission and the surrogate for G_2 is influenced by the operating environment.